Maths Calculation Policy

Love, Learn and Shine Together with Jesus

Written: September 2023

Date of Review: September 2024

Leader: Mr J Brooks

Calculation Policy

This Calculation Policy has been developed to support the effective implementation of the 2014 Primary National Curriculum. It should be looked at in conjunction with the Maths Policy.

At St. Matthew's Catholic Primary School, we implement the Mastery approach. At the centre of this approach is the belief that all children have the potential to succeed. Children should have access to the same curriculum content outlined in the National Curriculum programmes of study for their year group. The Mastery approach encourages depth before breadth, so that children become fluent in the fundamentals of mathematics and can apply their knowledge rapidly and accurately.

Purpose

This policy outlines the progression of calculation strategies from EYFS – Year 6, in line with the requirements of the 2014 Primary National Curriculum. It also supports teachers in identifying appropriate pictorial representations and concrete materials to help develop understanding.

The policy details the calculation strategies; teachers must plan opportunities for pupils to apply these; for example, when problem solving or when opportunities emerge elsewhere in the curriculum.

It is expected that children will be encouraged to build their fluency, problem solving and reasoning in all 4 operations by taking this approach:

Concrete– children should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.

Pictorial – alongside this, children should use pictorial representations. These representations can then be used to help reason and solve problems.

Abstract – both concrete and pictorial representations should support children's understanding of abstract methods; using numbers and key concepts with confidence.

True understanding of a mathematical concept may require going back and forth between these representations and using them alongside one another.

Bar Modelling

Bar modelling is used widely as an effective part of the Concrete, Pictorial, Abstract approach to the mastery of mathematics. Concrete materials are embedded alongside pictorial representations and abstract expressions to ensure procedural fluency and conceptual understanding are developed together.

Bar models are pictorial representations of problems or concepts that can be used for any of the 4 operations. They are also helpful for fractions, percentages, ratio and algebra. It is not a method for problem solving but does reveal the mathematical structure beneath the problem and mathematical relationships between its component parts. Bar models can help children decide which operations to use or to visualise problems. By using the bar method to visualise problems, pupils are able to tackle number or complex problems.

The purpose of this policy is to outline the progression of use of the bar model across St. Matthew's Catholic Primary School and will run alongside the Maths Policy.

Although the use of the bar model is not statutory, **it is expected that all staff will teach and model this approach as a method that the children can draw upon where and when required. To be proficient at using this model, children need to be introduced to it early in their education.**

We are using this model in line with the Concrete, Pictorial, Abstract (CPA) approach.

<u>A 5-step guide to bar modelling following the CPA approach</u>

<table>
<tr><td>1. Concrete resources (real objects)</td><td>
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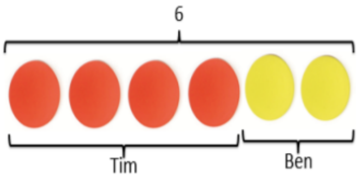
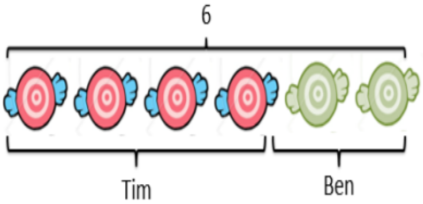
<tr><td>2. Substituted concrete resources (counters, cubes, buttons etc)</td><td>
</td></tr>
<tr><td>3. Pictorial representation</td><td>
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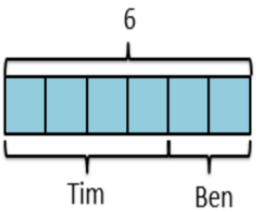
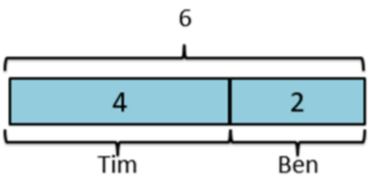
<tr><td>4. Discrete bar model</td><td>
</td></tr>
<tr><td>5. Rectangular bars</td><td>
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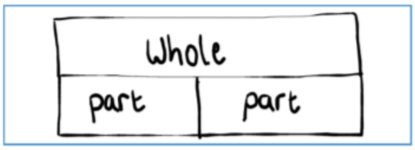
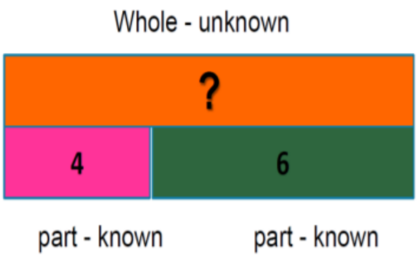
<u>Types of bar model</u>

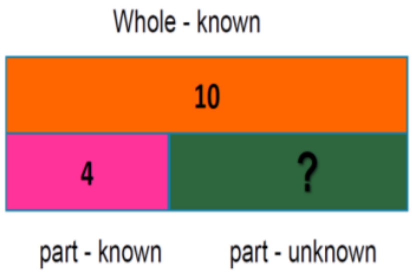
There are two types of bar model, ‘part/whole’ and ‘comparison’.

1. Part/Whole bar models

Part/whole bar models are made up of parts and wholes, where the whole represents the sum of the parts.

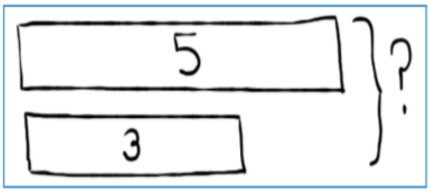


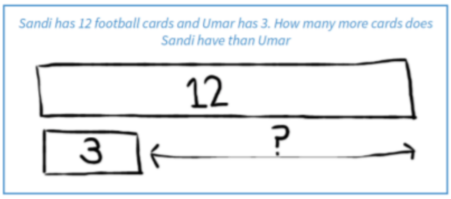




2. Comparison model

In a comparison model, two or more bars are drawn to help children to compare two or more amounts. Comparison models are particularly useful when finding differences between amounts, helping to reinforce the idea of using subtraction to find the difference.





For both the 'part/whole' and the 'comparison' models, the use of the question mark is important. If children consider where their answers should be on the diagram, (the unknown quantity marked '?'), then they are thinking about what the question is asking them to find out.

Although useful, bars don't need to be proportionally accurate although if a bar is representing 12 and another bar is representing 20, the 20 one should clearly be longer than the 12.

Importance of language when introducing bar models

The language of 'part' and 'whole' should be used consistently for a good success rate with bar models and the 'knowns' and 'unknowns' in different problem types must be emphasised. Scaffolding questions to ask, to help visualise the problem using a bar model:

- What do we know – parts or whole?
- What is the unknown – parts or whole?
- If I know this, therefore I know…
- Label the 'known' parts and/or the 'whole'
- Label the 'unknown' parts and/or the 'whole'

• Write the number sentence/equation

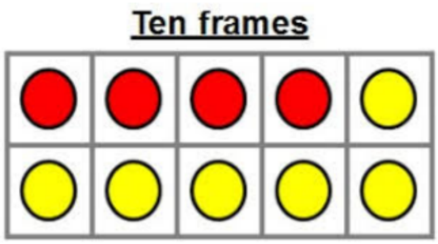
Ten Frames in EYFS

Ten frames help children develop number sense. In EYFS, children should have lots of opportunities to use ten frames, as it teaches them to subitise and is a precursor to addition and subtraction and bar models.

They are a highly effective way to teach the skills required to recognise and understand number patterns that are essential for operational fluency in maths, including the ability to add and subtract mentally, to see relationships between numbers and to see patterns. It also allows children to become familiar with 'parts' and 'wholes', 'knowns' and 'unknowns' before using the bar model.

They should be used to prompt different mental images of numbers and different mental strategies for manipulating these numbers.

It is important to follow the CPA approach when using ten frames.



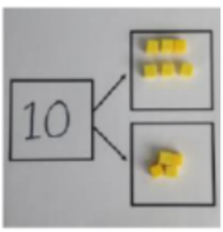
Part-Whole diagrams in EYFS and Y1

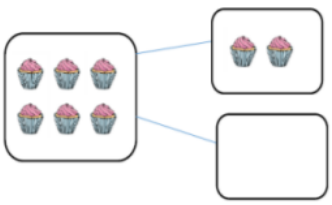
The part-whole model is the concept of how numbers can be split into parts. Children using this model will see the relationship between the whole number and the component parts, this helps learners make the connections between addition and subtraction.

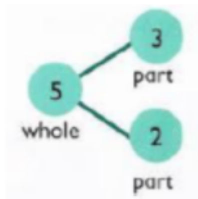
Part-whole reasoning also helps pupils to interpret, visualise and solve word problems.

Using the language of 'part' 'whole', 'known' and 'unknown', when using the part-whole diagram, is a pre-cursor to bar models.

The CPA approach will be followed when using the part-whole diagram.





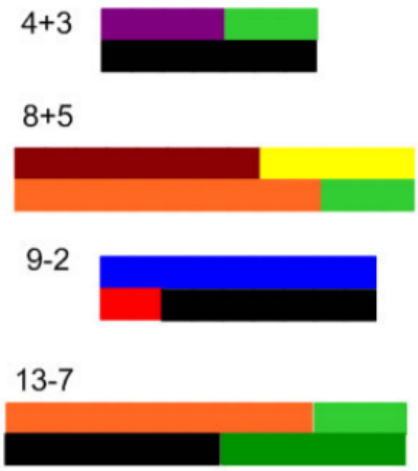


Cuisenaire Rods

All classes have a set of Cuisenaire rods and they can be used as an effective tool to develop children's understanding of the structure of mathematics from EYFS to Y6. They can be used for various topics such as fractions, ratio and algebra.

These are also a good tool for children to understand the structure of the bar model – what the bars represent, in terms of part, part, whole. Lots of practice should take place with these rods and once a numerical value is assigned to the rods, it supports children's understanding of calculations and the relationship of one number to another.

It is important that all children have the opportunity to access Cuisenaire rods to support conceptual understanding – particularly as a pre-requisite to and then, alongside bar modelling.



There is a clear focus on manipulatives and visual images to support understanding in every year group. Each new concept or calculation strategy is introduced using appropriate manipulatives. It is important that the children have access to a wide range of manipulatives in every year group, when and where applicable.

These include:

Addition	Subtraction	Multiplication	Division
100 square	100 square	Place value counters	Arrays
Number lines/tracks	Number lines/tracks	Dienes	Multiplication
Bead strings	Bead strings	Place value charts	squares
Straws	Straws	Arrays	100 square
Dienes	Dienes	Multiplication	Number lines
Place value cards	Counting stick	squares	Blank number lines
Place value dice	Place value dice	100 square	Counting stick
Place value counters	Place value cards	Number lines	Place value counters
Numicon	Place value counters	Blank number lines	Dienes
Multi-link cubes	Multi-link cubes	Counting stick	Numicon
Blank number lines	Blank number lines	Multi-link cubes	Counting Stick
Counting stick	Numicon	Numicon	Multi-link cubes
Cuisenaire rods	Cuisenaire rods	Counters	Counters
Ten frames	Ten frames	Cuisenaire rods	Cuisenaire rods
		Ten frames	Ten frames

Addition, Subtraction, Multiplication and Division

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods for all 4 operations.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.

Children are taught and acquire secure mental methods of calculation and one written method of calculation for the 4 operations, which they know they can rely on when mental methods are not appropriate.

Addition

This policy shows the stages of each written method for addition, each stage building towards a more refined method.

There are some key basic skills that children need to help with addition, which include:

• counting

• estimating

• recalling all addition pairs to 10, 20 and 100 (7 + 3 = 10, 17 + 3 = 20, 70 + 30 = 100)

• knowing number facts to 10 (6 + 2 = 8)

• adding mentally a series of one-digit numbers (5 + 8 + 4)

• adding multiples of 10 (60 + 70) or of 100 (600 + 700) using the related addition fact, 6 + 7,

and their knowledge of place value

• partitioning two-digit and three-digit numbers into multiples of 100, 10 and 1 in different

ways (432 into 400 + 30 + 2 and also into 300 + 120 + 12)

• understanding and using addition and subtraction as inverse operations

Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

• using inverse

• missing box questions

• using units of measure including money and time

• word problems

• open ended investigations

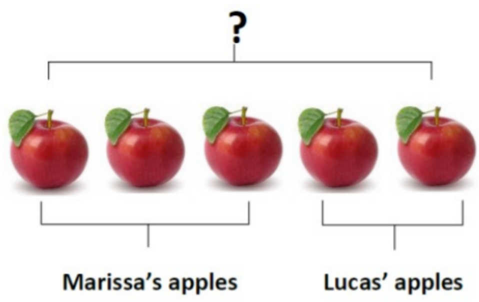
Using Bar Models in Addition

Example question for addition: Children will routinely come across calculations such as 3 + 2. Often, these calculations will be presented as word problems:

Marissa has 3 apples. Lucas has 2 apples. How many apples are there altogether?

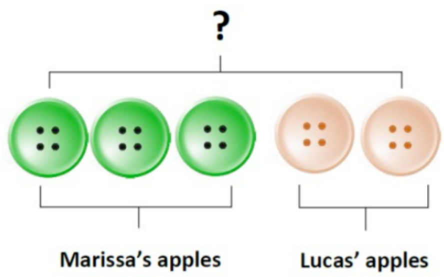
1. Using concrete resources

With addition, subtraction and multiplication, to help children fully understand the later stages of bar modelling, it is crucial they begin with concrete representations.



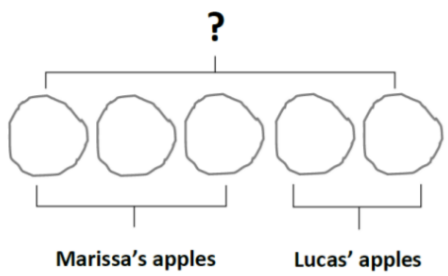
2. Using substituted concrete resources

Once they are used to the format and able to represent problems in this way themselves (assigning 'labels' verbally), the next stage is to replace the 'real' objects with resources that represent the object e.g. counters, cubes, multi-link, Cuisenaire rods.



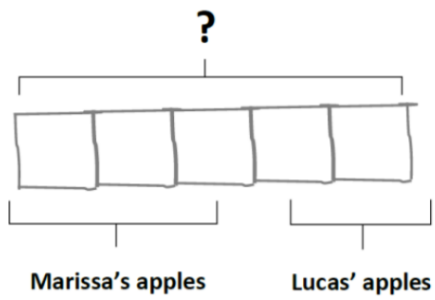
3. Pictorial representations

The next stage is to move away from the concrete to the pictorial.



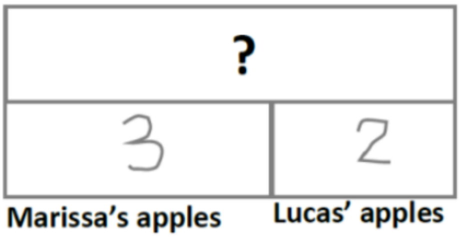
4. Discrete Bar Model

The next stage is to represent each object as part of a bar, in preparation for the final stage.



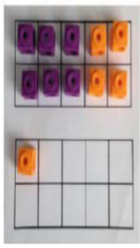
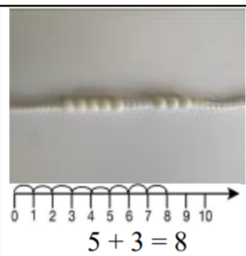
5. Rectangular Bars

The final stage stops the 1:1 representation and instead, each quantity is represented approximately as a rectangular bar.



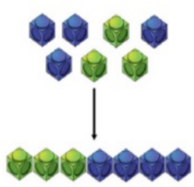
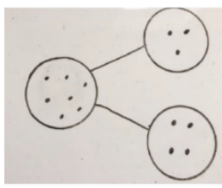
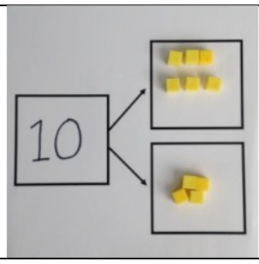
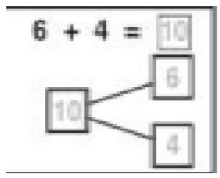
Stage 1: Practical (combining) and adding on (increasing)

Prior to recording addition steps on a number line, children will work practically with equipment where they are combining sets of objects. As they become more confident, this practical addition of sets of objects will be mirrored on a number line so that the two are being done together and children are adding on. This will prepare them for the abstract concept of adding numbers rather than objects.

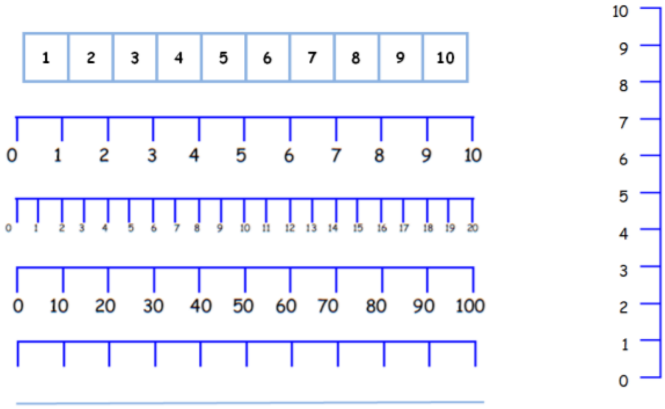
	 6 + 5 = 11 Start with the bigger number and use the smaller number to make 10.		

Part –part- whole

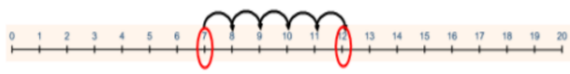
This model begins to develop the understanding of the commutativity of addition, as pupils become aware that the parts will make the whole in any order.

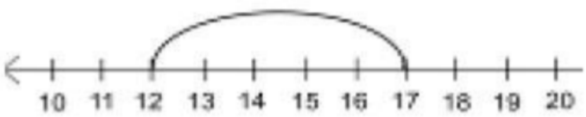
Stage 2: Number Tracks and Number Lines



Steps in addition can be recorded on a number line.



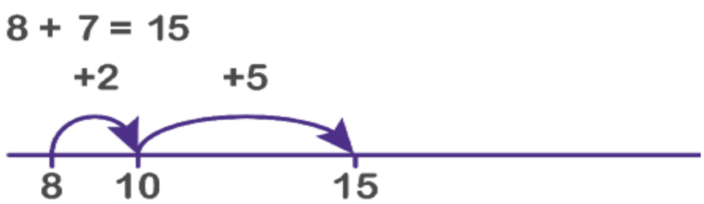
7 + 5 = 12



12 + 5 = 17

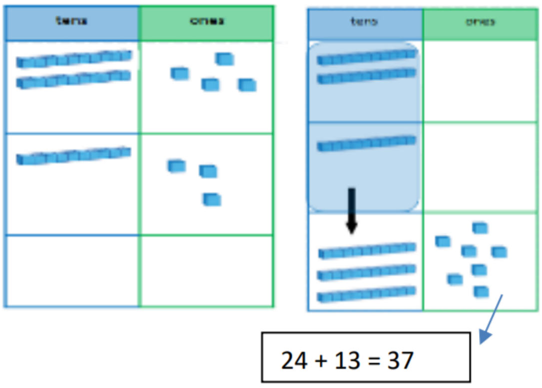
The steps often bridge through a multiple of 10 and, this is more efficient if children know how to partition 1-digit numbers.

In this example, 7 has been partitioned into 2 and 5 which makes bridging through 10 more efficient.

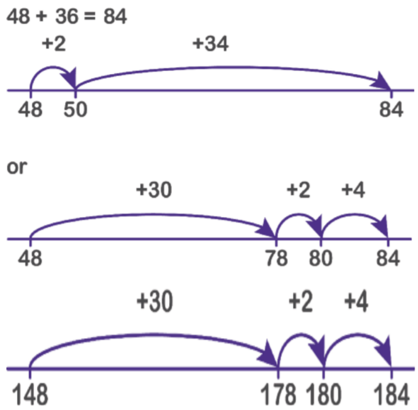


Stage 3 :Partitioning

Use partitioning to add at least two 2-digit numbers using concrete resources and/or a numbered number line and then progressing to an empty number line.



In these examples, the 6 in 36 has been partitioned into 2 and 4 which makes bridging through 10 more efficient.



With practice, children will need to record fewer jumps.

Stage 4 : Expanded columnar method

Partition both numbers into tens and ones or hundreds, tens and ones (using a place value grid makes this easier). Use manipulatives alongside the calculation.

This builds on children's mental maths skills of partitioning and recombining.

8 + 6 = 14
40 + 30 = 70
48 + 36 = 84

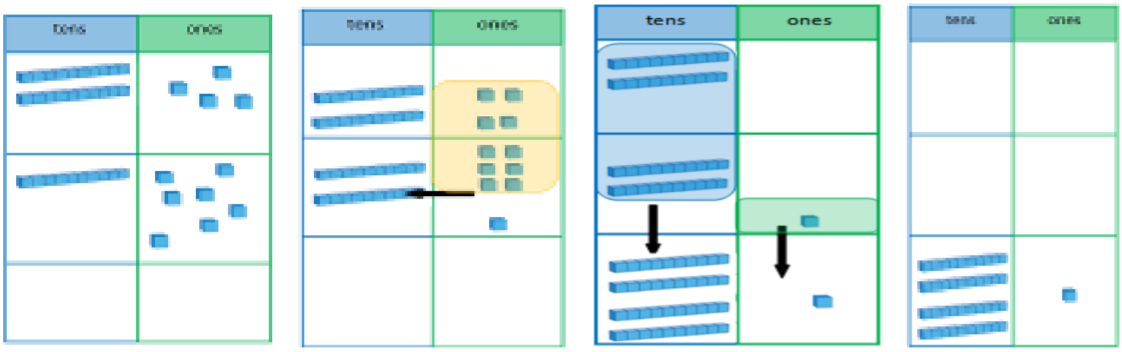
48 + 36 = 84

	40	8	
+	30	6	
	70	[1]4	84

148 + 36 = 184

	100	40	8	
+		30	6	
	100	70	[1]4	184

Leading to regrouping.



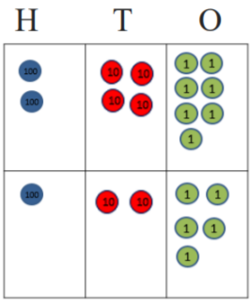
24 + 17 =

20 + 4

10 + 7

30 + 11 = 41

Using more abstract manipulatives as concrete understanding develops.



200 + 40 + 7

100 + 20 + 5

300 + 60 + 12 = 372

Leading to regrouping.

Stage 5: Efficient (column method)

Children move on to the formal column algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method. Children should be encouraged to estimate their answers first.

$$\begin{array}{r} 48 \\ +\ 36 \\ \hline 84 \\ \hline 1 \end{array} \qquad \begin{array}{r} 148 \\ +\ 36 \\ \hline 184 \\ \hline 1 \end{array} \qquad \begin{array}{r} 48.56 \\ +\ 32.23 \\ \hline 80.79 \\ \hline 1 \end{array}$$

Column addition remains efficient when used with larger whole numbers or decimals, and when adding more than two numbers. Once learned, the method is quick and reliable.

Subtraction

This policy shows the stages of each written method for subtraction, each stage building towards a more refined method.

There are some key basic skills that children need to help with subtraction, which include:

• counting

• estimating

• recalling all addition pairs to 10, 20 and 100 along with their inverses (7 + 3 = 10, 10 – 3 = 7, 17 + 3 = 20, 20 – 3 = 17, 70 + 30 = 100, 100 – 30 = 70)

• knowing number facts to 10 and their inverses (6 + 2 = 8, 8 - 2 = 6)

• subtracting multiples of 10 (160 - 70) using the related subtraction fact, 16 - 7, and their knowledge of place value

• partitioning two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways (432 into 400 + 30 + 2 and also into 300 + 120 + 12)

• understanding and using subtraction and addition as inverse operations

Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

• using inverse

• missing box questions

• using units of measure including money and time

• word problems

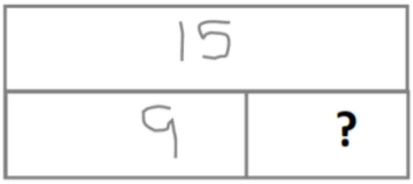
• open ended investigations

Using Bar Models in Subtraction

The same concrete to pictorial stages can be applied to subtraction. The method can be taught with the 'part/whole' bar model or the 'comparison' bar model.

Part/Whole model

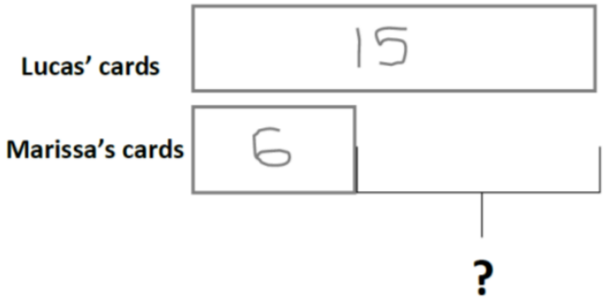
Lucas has 15 playing cards.
He gives 9 to his sister. How many cards does he have left?



or

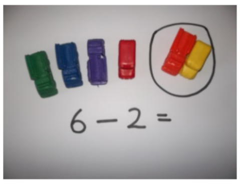
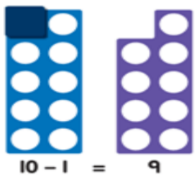
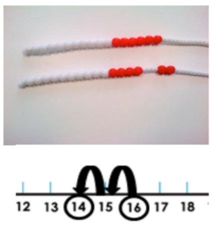
Comparison bar model

Lucas has 15 playing cards. Marissa has 6 playing cards. How many more playing cards does Lucas have than Marissa?

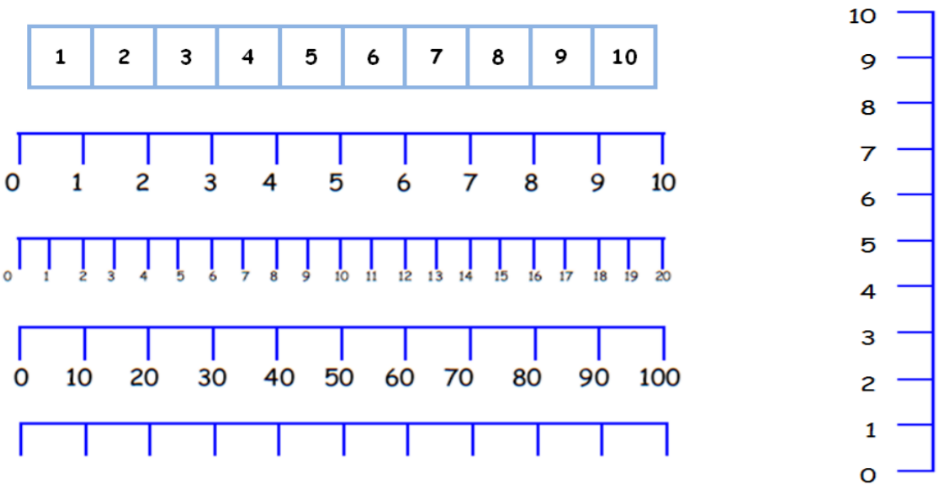


Stage 1: Practical (taking away)

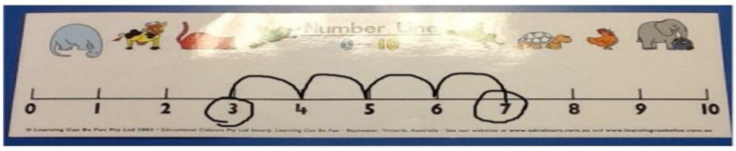
Prior to recording subtraction steps on a number line, children will work practically with equipment where they are 'taking away' a small group from a larger set of objects. As they become more confident, this practical subtraction will be mirrored on a number line so that the two are being done together. This will prepare them for the abstract concept of subtracting numbers rather than objects.

	 20 – 4 = 16		 16 – 2 = 14

Stage 2: Number Tracks and Number Lines (Partitioning)

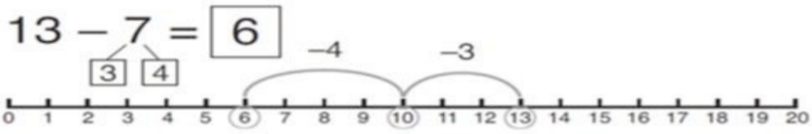


<u>Counting back</u>

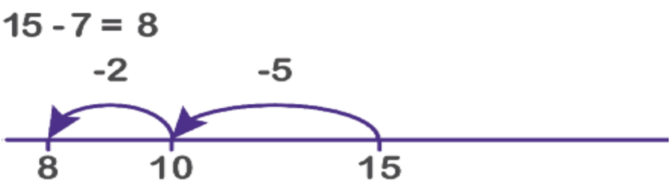


7 - 4 = 3

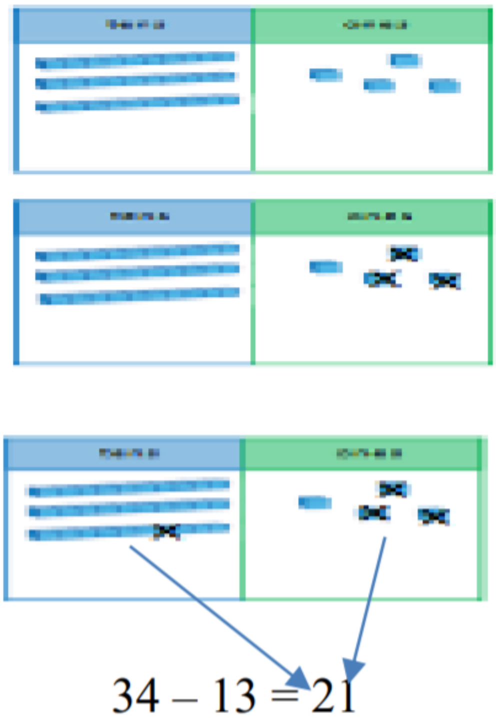
Steps in subtraction can be recorded from right to left on a number line. The steps often bridge through a multiple of 10 and, this is more efficient if children know how to partition 1- digit numbers.



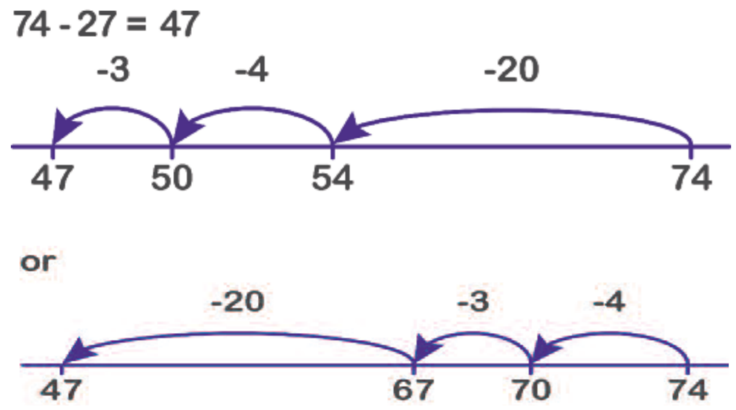
In this example on a blank number line, 7 has been partitioned into 2 and 5 which makes bridging through 10 more efficient.

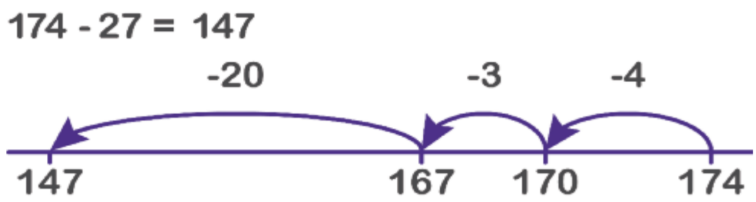


Use partitioning to subtract two 2-digit numbers using concrete resources and/or a numbered number line and then progressing to an empty number line.



In these examples, 27 has been partitioned into tens and ones then the 7 in 27 has been partitioned into 3 and 4 which makes bridging through 10 more efficient.



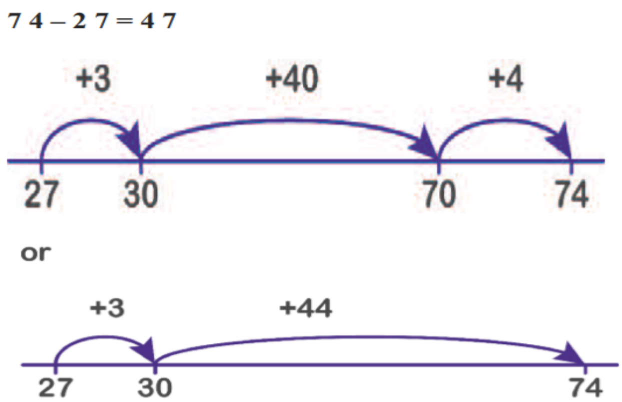


With practice, children will need to record fewer jumps.

<u>Find the difference (introduced after 'counting back')</u>

This strategy should be used when the language used is 'find the difference', 'difference between' and 'distance between'.

Steps in subtraction can be recorded from left to right on a number line. The steps often bridge through a multiple of 10. When carrying out money calculations that involve finding change or when calculating time duration, children should use this method.



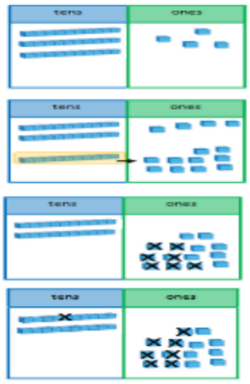
With practice, children will need to record fewer jumps.

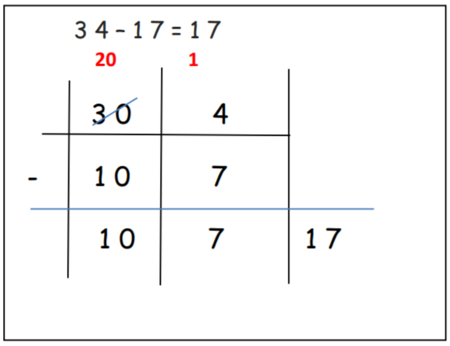
They will decide whether to count back or forwards, seeing both as 'finding the difference'. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12 or 86 – 77.

Stage 3: Expanded columnar method

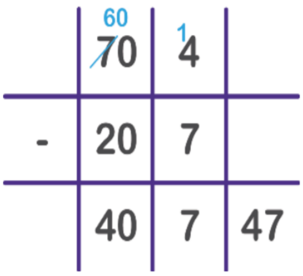
Partition both numbers into tens and ones or hundreds, tens and ones (using a place value grid makes this easier). Use manipulatives alongside the calculation.

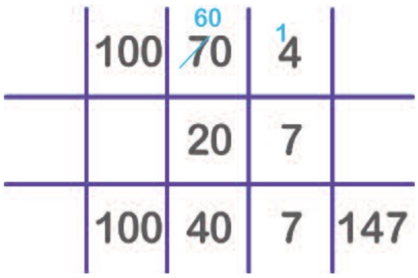
34 – 17 = 17



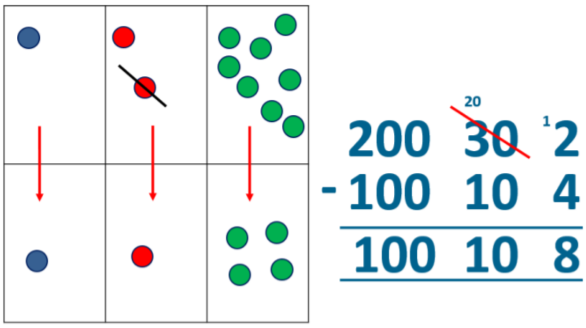


In this example, to subtract 7 ones from 4 ones, we need to regroup 10 ones for a ten. We now can subtract 7 ones from 14 ones.





Using more abstract manipulatives as concrete understanding develops.



Stage 4 Efficient (column method)

Children move on to the formal column algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method. Children should be encouraged to estimate their answers first.

$\overset{6}{\cancel{7}}{}^{1}4$	$1\overset{6}{\cancel{7}}{}^{1}4$	48.56
- 27	- 27	- 32.23
47	147	16.33

Column subtraction remains efficient when used with larger whole numbers or decimals, once learned, the method is quick and reliable.

Multiplication

This policy shows the stages of each written method for multiplication, each stage building towards a more refined method.

There are some key basic skills that children need to help with multiplication, which include:

• counting

• estimating

• understanding multiplication as repeated addition

• recalling all multiplication facts to 12 × 12

• partitioning numbers into multiples of one hundred, ten and one

• working out products (70 × 5, 70 × 50, 700 × 5, 700 × 50, 0.5 x 7, 0.05 x 7) using the

related fact 7 × 5 and their knowledge of place value

• adding two or more single-digit numbers mentally

• adding multiples of 10 (60 + 70) or of 100 (600 + 700) using the related addition fact, 6 + 7,

and their knowledge of place value

• adding combinations of whole numbers

• understanding and using division and multiplication as inverse operations

Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their

skills are broadened through their use and application in a range of contexts, these include:

• using inverse

• missing box questions

• using units of measure including money and time

• word problems

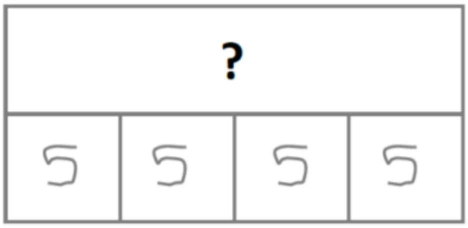
• open ended investigations

Using Bar Models in Multiplication

Bar models of multiplication start with the same ‘real’ and ‘substituted’ concrete resource stages as addition and subtraction. Then moves to its final stage, drawing rectangular bars to represent each group (it is necessary to understand multiplication as repeated addition).

Example question for multiplication: Children will routinely come across calculations such as 5 x 4. Often, these calculations will be presented as word problems:

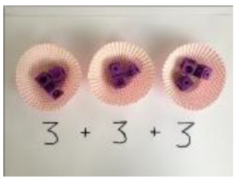
Marissa buys 4 boxes of cookies. Each box contains 5 cookies. How many cookies does Marissa have?



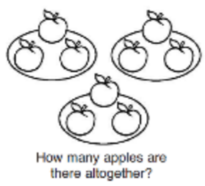
Stage 1: Practical (repeated addition)

Children will work practically with equipment grouping objects to see multiplication as repeated addition.

As they become more confident, this practical grouping of objects will be mirrored on a number line using the vocabulary ‘lots of’, ‘groups of’, ‘how many lots’, ‘how many times’ so that the two are being done together. This will prepare them for the abstract concept of multiplying numbers rather than objects.

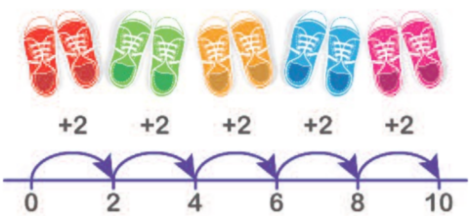


$3 + 3 + 3 = 9$



$3 + 3 + 3 = 9$

$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$



This image can be expressed as:

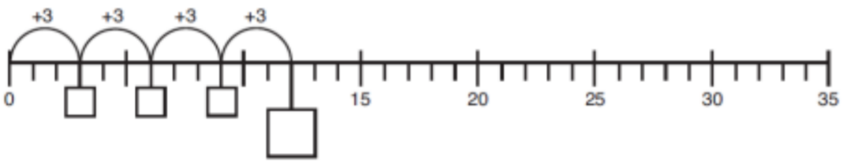
• 2 multiplied by 5

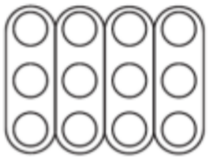
• two, five times

• 5 groups of 2

• 5 lots of 2

• 5 jumps of 2 on a number line

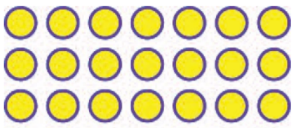




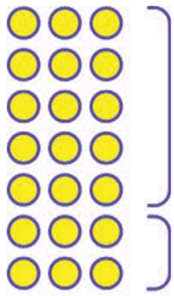
$4 \times 3 = \square$

Stage 2: Practical and pictorial arrays (towards grid method)

Children use arrays to demonstrate their understanding of commutativity for multiplication facts. Children can use counters, cubes, pictures to show the arrays.



7 x 3 = 21



3 x 7 = 21

Children use their knowledge of known multiplication tables
This 3 x 7 array can also be seen as
3 x 5 add 3 x 2

Stage 3: Partitioning (grid method)

Children will use concrete resources to develop conceptual understanding of the compact method introduced in Year 4. These should be used alongside the grid method.

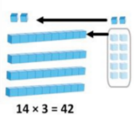
×	10	2
3		

×	10	2
3	30	6

3 x 12 = 36

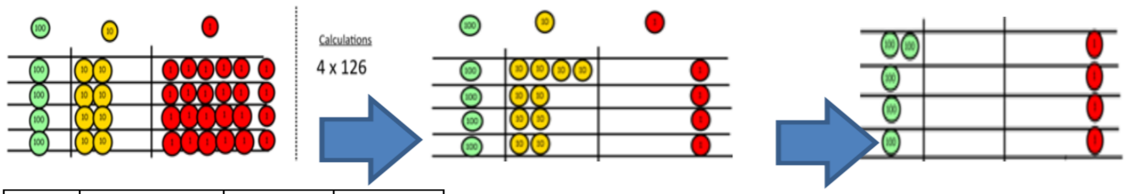
×	10	4
3	30	12

×	10	4
3	30	12



3 x 1 4 = 4 2

Using more abstract manipulatives as concrete understanding develops. Demonstrate the regrouping with the manipulatives. Use the grid method alongside.



x	100	20	6
4	400	80	24

$126 \text{ X } 4 = 5\,0\,4$

Moving on to abstract.

24 x 3 = 72

x	20	4	
3	60	12	72

24 x 32 = 768

x	20	4	
30	600	120	720
2	40	8	48
			768

Stage 4: Short (column method)

Use concrete resources if needed to demonstrate multiplying numbers by one digit using the compact short multiplication method.

24 x 3 = 72

$$\begin{array}{r} 24 \\ \times\ \ 3 \\ \hline 72 \\ \hline {\scriptstyle 1} \end{array}$$

1241 x 3 = 3723

$$\begin{array}{r} 1241 \\ \times\ \ \ \ \ 3 \\ \hline 3723 \\ \hline {\scriptstyle 1\ \ \ } \end{array}$$

Stage 5: Long (column method)

Reinforce the connection between the grid method to multiply numbers up to 4 digits by two digit using long multiplication.

$$\begin{array}{r} 24 \\ \times\ \ 32 \\ \hline 48 \\ 720 \\ \hline {}^{1}768 \end{array} \qquad\qquad \begin{array}{r} {}^{1\ 1} \\ 1245 \\ \times\qquad 13 \\ \hline 3735 \\ 12450 \\ \hline 16185 \\ \hline 1 \end{array}$$

In the examples given, it is also correct to multiply starting with the tens digit (ie multiplying by the most significant digit first), however as school policy, we will teach starting from the ones digit.

Division

This policy shows the stages of each written method for division, each stage building towards a more refined method.

There are some key basic skills that children need to help with division, which include:

• counting

• estimating

• understanding division as repeated subtraction

• partitioning two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways (432 into 400 + 30 + 2 and also into 300 + 120 + 12)

• recalling multiplication and division facts to 12 × 12

• recognising multiples of one-digit numbers and dividing multiples of 10 or 100 by a single digit number using their knowledge of division facts and place value

• knowing how to find a remainder working mentally, for example, find the remainder when

48 is divided by 5

• understanding and using division and multiplication as inverse operations

Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

• using inverse

• missing box questions

• using units of measure including money and time

• word problems

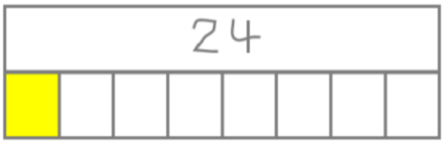
• open ended investigations

Using Bar Models in Division

For division it is recommended that children remain grouping and sharing until the final stage of bar modelling is understood. Then word problems can be introduced, using the final stage of rectangular bars.

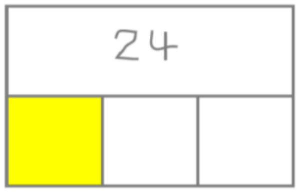
Sharing problem

Lucas has 24 lollies. He wants to share them into 8 party bags for his friends. How many lollies will go into each party bag?



Grouping problem

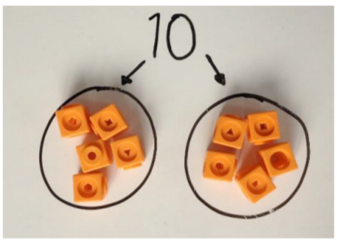
Lucas has 24 lollies for his party friends. He wants each friend to have 3 lollies. How many friends can he invite to his party?

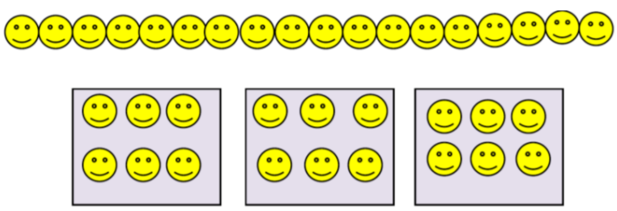


Stage 1: Practical (sharing)

Children will work practically with equipment sharing objects one to one.

12 cakes are shared equally between 3 people.



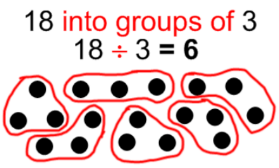


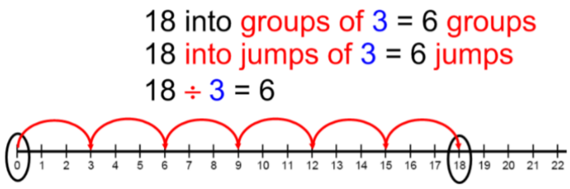
10 cubes between 2 people? Can you share 18 smiley stickers between 3 people?

Stage 2: Number Lines (grouping)

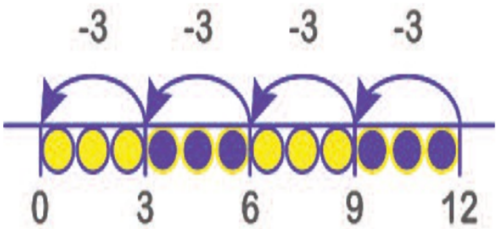
Children will move from sharing objects practically to grouping them, this will be mirrored on a number line, working from right to left so that they see division as repeated subtraction.

This will prepare them for the abstract concept of dividing numbers rather than objects.





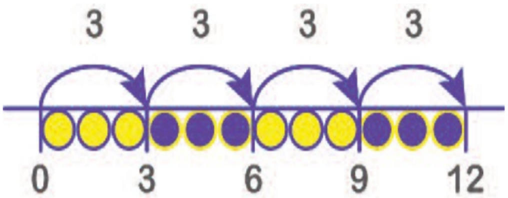
Each cake box holds 3 cakes, if I have 12 cakes, how many cake boxes will I need?



How many times can I subtract 3 from 12?

Using their knowledge of the inverse relationship between multiplication and division,

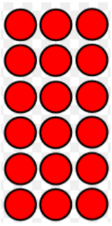
children can use their multiplication tables when grouping on a number line, working from left to right.



How many groups of 3 are there in 12?

First without and then with remainders and ensuring that divisors offer an appropriate level of challenge.

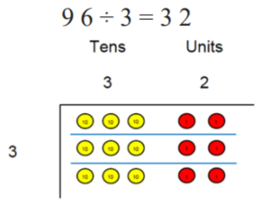
Reinforce division through the use of arrays.

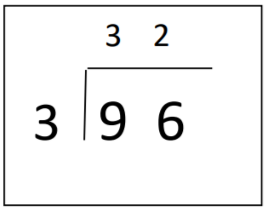


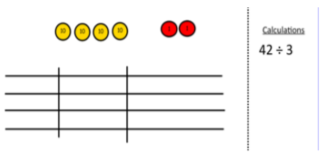
1 8 ÷ 3 = 6
1 8 ÷ 6 = 3

Stage 3: Short Division

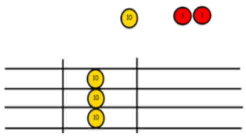
Use place value counters to divide using the bus stop method alongside.



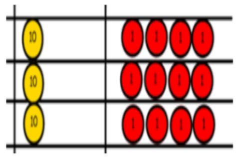




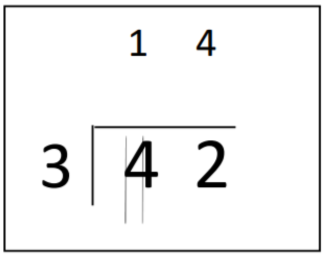
Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



We regroup this ten for ten ones and then share the ones equally among the groups.



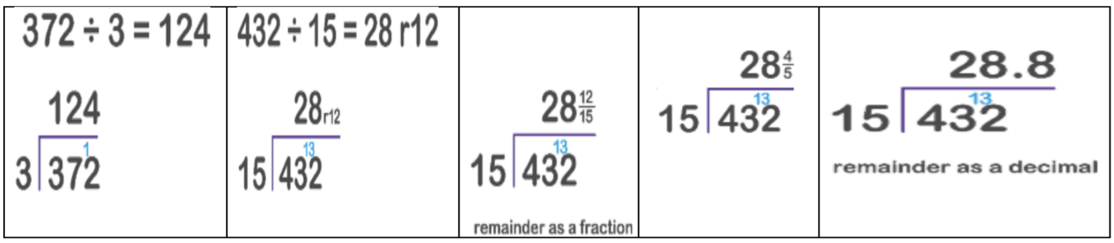
We look how much is in 1 group, it is 14.



Begin with divisions that divide equally with no remainder.

Move onto divisions with a remainder.

Move on to representing the remainder as a fraction, then a decimal.



Stage 4: Long Division

$$432 \div 15 = 28\ r12$$

$$\begin{array}{rl}
 & 28_{r12} \\
15\,\big) & 432 \\
 & \underline{300} \quad 15\times20 \\
 & 132 \\
 & \underline{120} \quad 15\times8 \\
 & 12
\end{array}$$

Fraction remainder – 12/15 = 4/5

Decimal remainder – 12 ÷ 15 = 0.8

$$\begin{array}{rl}
 & 28.8 \\
15\,\big) & 432.0 \\
 & \underline{30}\downarrow \\
 & 132 \\
 & \underline{120} \\
 & 12\,0 \\
 & \underline{12\,0} \\
 & 0
\end{array}
\qquad
\begin{array}{rl}
 & 28\tfrac{4}{5} \\
15\,\big) & 432 \\
 & \underline{30}\downarrow \\
 & 132 \\
 & \underline{120} \\
 & 12 \\
 & 12
\end{array}$$

$(12 \div 15 = 0.8)$ $(0.8 = \tfrac{4}{5})$

remainder as a decimal   remainder as a fraction

Children write the multiples of the divisor.

15	75	135
30	90	
45	105	
60	120	

$$432 \div 15 =$$

$$\begin{array}{rl}
 & 2\ 8\ \ r\ 12 \\
15\,\big) & 432 \\
 & 13
\end{array}$$

Fraction remainder – 12/15 = 4/5
Decimal remainder – 12 ÷ 15 = 0.8

Children write the multiples of the divisor.

15	75	135
30	90	
45	105	
60	120	

The Calculation Sequence – Applying the Skills

The Sequence	Prompts
Provide an estimate for the calculation	Using knowledge of number and the number system, rounding and approximating, make a reasonable estimate.
Teach the calculation skill	What is the objective you are teaching? Include example questions, increasing in complexity, for both operations.
Ensure you have taught the inverse	Plan example questions, increasing in complexity. Ensure methods used are in line with school calculation policy. Check that children understand that inverse can also be used to check calculations
Devise similar calculations but include units	Which units do you need to include? Check the measures applicable to your year group for length, weight, capacity, money and time.
Complete missing box questions	Include units in these questions as above. The box may cover single digits or an entire number. Vary the position of the missing box within the calculation
Complete word problems, 1 and 2 step, including units	Write problems, ensuring the numbers are sized correctly in line with the objective and that units are also used.
Provide opportunities for open ended investigations	Plan example questions and investigations. Ensure children are working with the correct operations, appropriate size of numbers and use of units for context.

Progression of Addition

Typical Calculations	Suitable Methods
O + O (no bridging) (then bridging 10) TO + O	Aggregation (Counting back) Augmentation (count on) Practical Number line
O+O TO + O (to 20 including zero)	Practical Number line
TO + O TO + multiples of 10 TO + TO O + O + O	Practical Number Line Partitioning on a number line Expanded columnar
HTO + O HTO + TO HTO + HTO	Number line Expanded columnar Column
THTO + HTO	Expanded columnar
THTO + THTO	Column
THTO.t + THTO.t THTO.th + THTO.th Add whole numbers with more than 4 digits	Expanded columnar Column
THTO.thth + THTO.thth Add whole numbers with more than 4 digits	Column

Progression of Subtraction

Typical Calculations	Suitable Methods
1.O – O 2.TO - O	1.no bridging 10 2. no bridging 3. bridging 10 **Practical** (counting out, counting back from, count back to (counts the fingers) **Number line**
O-O TO - O (to 20 including zero)	Practical Number line
TO - O TO - multiples of 10 TO - TO O - O - O	Practical Number line Expanded columnar
HTO - O HTO - TO HTO - HTO	Number line Expanded columnar Column
THTO - HTO	Expanded columnar
THTO - THTO	Column
THTO.t - THTO.t THTO.th - THTO.th Subtract whole numbers with more than 4 digits	Expanded columnar Column
THTO.tht - THTO.tht Subtract whole numbers with more than 4 digits	Column

Progression of Multiplication

Typical Calculations	Suitable Methods
Doubling as repeated addition	Practical (repeated addition)
O x O	Practical (repeated addition) Grouping on a number line (arrays)
O x O	Practical (repeated addition) Practical and pictorial arrays Grouping on a number line
TO x O	Grouping on a number line progressing into Expanded (grid) and into Short
TO x O HTO x O	Expanded (grid) progressing into Short
HTO x O THTO x O TO x TO HTO X TO	Expanded (grid) progressing into Short Expanded (grid) progressing into Long
THTO x O	Short
TO x TO HTO x TO THTO x TO O.t x O O.th x O O.t x TO O.th x TO	Expanded (grid) progressing into Long Long Expanded (grid) progressing into Short Expanded (grid) progressing into Long

Progression of Division

Typical Calculations	Suitable Methods
Talk about halving Sharing into equal groups	Practical sharing into equal groups
O ÷ O TO ÷ O	Practical sharing Number line grouping
O ÷ O TO ÷ O	Practical sharing Number line grouping
TO ÷ O	Grouping on a number line progressing into Short
TO ÷ O HTO ÷ O	Grouping on a number line progressing into Short Short (remainders to be expressed as r)
HTO ÷ O THTO ÷ O	Short (remainders to be expressed as r,
	then as a fraction and as a decimal)
THTO ÷ O HTO ÷ TO THTO ÷ TO O.th ÷ O TO.th ÷ O HTO.th ÷ O THTO.th ÷ O	Short (remainders to be expressed as r, then as a fraction and as a decimal) Long (remainders to be expressed as r, then as a fraction and as a decimal) Short (remainders to be expressed as a decimal)